



WISSENSCHAFTSZENTRUM BERLIN  
FÜR SOZIALFORSCHUNG

SOCIAL SCIENCE RESEARCH  
CENTER BERLIN

**discussion papers**

FS IV 02 – 30

**A Note on the Determinants of Labour Share Movements**

Sebastian Kessing

Freie Universität Berlin

December 2002

ISSN Nr. 0722 - 6748

**Forschungsschwerpunkt  
Markt und politische Ökonomie**

**Research Area  
Markets and Political Economy**

Zitierweise/Citation:

Sebastian Kessing, **A Note on the Determinants of Labour Share Movements**, Discussion Paper FS IV 02-30, Wissenschaftszentrum Berlin, 2002.

Wissenschaftszentrum Berlin für Sozialforschung gGmbH,  
Reichpietschufer 50, 10785 Berlin, Tel. (030) 2 54 91 – 0  
Internet: [www.wz-berlin.de](http://www.wz-berlin.de)

## ABSTRACT

### **A Note on the Determinants of Labour Share Movements**

by Sebastian Kessing\*

Adjustment costs cause movements of the labour share if the economy experiences demand or wage shocks. With linear adjustment costs and Cobb-Douglas technology, these movements are independent of the size of these shocks and depend only on the size of the adjustment costs.

*Keywords: Labour share, adjustment costs, labour demand*

*JEL classification numbers: D33, J63*

## ZUSAMMENFASSUNG

### **Determinanten von Lohnquotenschwankungen**

Anpassungskosten auf dem Arbeitsmarkt verursachen Schwankungen der Lohnquote, wenn eine Volkswirtschaft Lohn- oder Nachfrageschocks ausgesetzt ist. Für den Fall einer Cobb-Douglas Produktionstechnologie wird gezeigt, dass die induzierten Schwankungen nicht von der Größe der Lohn- oder Nachfrageschocks abhängen, sondern nur von der Höhe der Anpassungskosten.

---

\* I would like to thank Giuseppe Bertola, Dan Hamermesh, Winfried Koeniger and Jarkko Turunen for comments. Financial support from DAAD is gratefully acknowledged.

# 1 Introduction

Time series properties of, as well as cross-country differences in, labour share movements have recently been recongnized as a key magnitude for understanding the interaction between macroeconomic shocks, institutions and unemployment, see Blanchard (1997, 1998) and Caballero and Hammour (1998). Bentolila and St. Paul (1999) have shown empirically, that adjustment costs are the single most important factor affecting labour share movements. This note considers how shocks, either to general business conditions (a demand shock, for example) or to wages, translate into labour share movements in the case of linear adjustment costs. Two neutrality results with respect to the size of the shocks affecting business conditions or wages are derived. If technology is Cobb-Douglas, the size of labour share fluctuations may only depend on the size of adjustment costs.

## 2 Factor shares in a Markov chain model

Consider the simple stochastic labour demand model of Bertola (1990). A representative risk-neutral firm's dynamic labour demand problem is given

by

$$\text{Max}_{\{L_i\}} E_t \left\{ \sum_{k=0}^{\infty} \left( \frac{1}{1+r} \right)^k [R(Z_{t+k}, L_{t+k}) - w_{t+k}L_{t+k} - C(L_{t+k} - L_{t+k-1})] \right\}. \quad (1)$$

$R_i(Z_i, L_i)$  denotes the firm's one period revenue, as a function of the amount of labour employed  $L_i$  and the prevailing business conditions  $Z_i$ . It is assumed either that business conditions follow a two state (good,  $i = g$ , and bad,  $i = b$ ) Markov chain (in which case the wage is assumed to be constant) or that the wage rate  $w_i$  follows a two state (high,  $i = h$ , and low,  $i = l$ ) Markov chain, in which case  $Z_i$  is constant and set to unity. Letting  $H$  and  $F$  represent the given costs per hired and fired worker respectively, the firm's asymmetric linear costs of adjusting its labour force are

$$C(L_i - L_{i-1}) = \begin{cases} H(L_i - L_{i-1}) & \text{if } L_i - L_{i-1} > 0 \\ -F(L_i - L_{i-1}) & \text{if } L_i - L_{i-1} < 0. \end{cases} \quad (2)$$

Define  $V_t \equiv E_t \left\{ \sum_{k=0}^{\infty} \left( \frac{1}{1+r} \right)^k M_{t+k}(Z_{t+k}, L_{t+k}) - w_{t+k} \right\}$  as the shadow product of labor where  $M_i(Z_i, L_i) \equiv \frac{\partial R_i(Z_i, L_i)}{\partial L_i}$  is the marginal revenue product of labor (MRPL). The first order conditions of the firm's problem are given as

$$-F \leq V_t \leq H \text{ always,} \quad (3)$$

$$V_t = H, \text{ if } L_t - L_{t-1} > 0 \text{ and } V_t = -F, \text{ if } L_t - L_{t-1} < 0. \quad (4)$$

The firm's optimal policy is to hire either when business conditions improve or when wages go down and to fire either when times turn bad or when wages go up. When conditions stay the same, inaction is optimal, since there are no voluntary quits. Hence, if adjustment costs are not prohibitively high, employment itself follows a Markov chain. From (4) it must be that

$$V_t = M(Z_g, L_g) - w_g + \frac{1}{1+r} E[V_{t+1}] = H \quad (5)$$

when times turn good. Substituting  $E[V_{t+1}] = p_g H + (1 - p_g)(-F)$ , where  $p_g$  is the probability of good times remaining good next period, gives

$$w_g = M(Z_g, L_g) - \frac{1}{1+r}(1 - p_g)(H + F) - \frac{r}{1+r}H. \quad (6)$$

Analogous equations can be derived for bad times and for the case in which the wage is fluctuating. These equations show the wedges that are driven between wages and the MRPL by the presence of adjustment costs and which cause the labour share to vary. They implicitly define optimum labour demand as  $L_i = L_i(Z_i, w_i, H, F, p_i, r)$ . Comparative statics reveal that everything that increases the wedges  $Q_j \equiv |M_j - w_j|$ ,  $j = g, l$ , reduces labour demand and everything that increases the wedges,  $Q_k \equiv |M_k - w_k|$ ,  $k = b, h$ , increases labour demand in these states. The labour share in state

$i$ ,  $S_i$ , is given by

$$S_i = \frac{w_i L_i(Z_i, w_i, H(w_i), F(w_i), p_i, r)}{R_i(Z_i, L_i(Z_i, w_i, H(w_i), F(w_i), p_i, r))}, \quad i = g, b, h, l. \quad (7)$$

The effect of adjustment costs on the labour share is given in

**Proposition 1** *The labour share is unambiguously increased in bad times and in times of high wages, and reduced in good times and in times with low wages, by the presence of adjustment costs.*

**Proof:** The effect of an increase in employment for fixed  $Z$  and  $w$  equals

$$\frac{\partial S_i}{\partial L_i} = \frac{w(R_i - \frac{\partial R_i}{\partial L_i} L_i)}{R_i^2} > 0. \quad (8)$$

Since adjustment costs increase labour demand in bad times and with high wages, but reduce it in good times or with low wages, the labour share will be raised in bad times, or with low wages, and reduced in good times, or with high wages.  $\square$

In the case of Cobb-Douglas technology with multiplicative shocks revenue equals  $R(Z_i, L_i) = \frac{1}{1-\beta} Z_i L_i^{1-\beta}$  and the following neutrality result holds:

**Proposition 2** *With Cobb-Douglas revenue and multiplicative shocks, the size of labour share fluctuations is invariant with respect to the size of these shocks, as long as adjustment costs are not prohibitively high.*

**Proof:** From (7) it follows that in general

$$\frac{\partial S_i}{\partial Z_i} = \frac{wL_i}{R_i Z_i} \left( \frac{\partial L_i}{\partial Z_i} \frac{Z_i}{L_i} - \frac{\partial R_i}{\partial Z_i} \frac{Z_i}{R_i} \right), \quad i = g, b. \quad (9)$$

Optimal labour demands with adjustment costs equal  $L_g = [w + Q_g]^{-\frac{1}{\beta}} Z_g^{\frac{1}{\beta}}$  and  $L_b = [w - Q_b]^{-\frac{1}{\beta}} Z_b^{\frac{1}{\beta}}$ . Then revenues are  $R_b = \frac{1}{1-\beta} Z_b \left( [w - Q_b]^{-\frac{1}{\beta}} Z_b^{\frac{1}{\beta}} \right)^{1-\beta}$  and  $R_g = \frac{1}{1-\beta} Z_g \left( [w + Q_g]^{-\frac{1}{\beta}} Z_g^{\frac{1}{\beta}} \right)^{1-\beta}$ . Taking logs and differentiating yields  $\frac{\partial L_i}{\partial Z_i} \frac{Z_i}{L_i} = \frac{1}{\beta}, i = g, b$  and  $\frac{\partial R_i}{\partial Z_i} \frac{Z_i}{R_i} = \frac{1}{\beta}, i = g, b$ . Thus,  $\frac{\partial S_i}{\partial Z_i} = 0, i = g, b$ , due to (9).□

As is evident from (9), the effect on the size of labor share movements depends on the relative importance of labour demand and revenue elasticities. In the case of Cobb-Douglas these exactly balance.

When labour share movements are due to fluctuations in wages an analogous result can be derived if adjustment costs are proportional to wages. This is not implausible for severance payments and red tape costs, which are typically very labour intensive:

**Proposition 3** *If hiring and firing costs are proportional to wages and technology is Cobb-Douglas, the size of labour share fluctuations caused by wage fluctuations is invariant to the size of these fluctuations, as long as adjustment costs are not prohibitively high.*



**Proof:** With proportional adjustment costs  $H = cw$  and  $F = bw$  the wedges become  $Q_l = \frac{w(1-p_l)(c+b)}{1+r} + \frac{wrc}{1+r}$  and  $Q_h = \frac{w(1-p_h)(b+c)}{1+r} + \frac{wrb}{1+r}$ . The labour shares equal

$$S_l = \frac{w_l [w_l + Q_l]^{-\frac{1}{\beta}}}{\frac{1}{1-\beta} [w_l + Q_l]^{1-\beta}} = \frac{(1-\beta)}{\left(1 + \frac{(1-p_l)(c+b)}{1+r} + \frac{rc}{1+r}\right)}, \quad (10)$$

$$S_h = \frac{w_h [w_h - Q_h]^{-\frac{1}{\beta}}}{\frac{1}{1-\beta} [w_h - Q_h]^{1-\beta}} = \frac{(1-\beta)}{\left(1 - \frac{(1-p_h)(c+b)}{1+r} - \frac{rb}{1+r}\right)}.$$

Obviously, the labour share does not depend on wages in both states. Thus,

$$\frac{\partial S_i}{\partial w_i} = 0, i = h, l. \square$$

If adjustment costs are proportional their relative importance remains constant, which translates into a constant size of labour share movements.

### 3 Conclusion

If production is Cobb-Douglas and adjustment costs are linear, factor share movements do not depend on the size of the demand shocks hitting the economy, but only the size of the adjustment costs. Similarly, if production is Cobb-Douglas and adjustment costs are linear and proportional to wages, factor share fluctuations do not depend on the size of wage shocks, but only on the size of adjustment costs. Consequently, the size of adjustment costs, and the labour market institutions that determine them, are more important

for labour share movements than the size of, and differences in, demand and wage shocks.

## References

- [1] Bentolila, Samuel and Gilles Saint-Paul (1999), "Explaining Movements in the Labour Share", CEMFI Working Paper 9905.
- [2] Bertola, Giuseppe (1990), "Job Security, Employment and Wages", *European Economic Review*, 34, 851-886.
- [3] Blanchard, Olivier (1998), "Revisiting European Unemployment: Unemployment, Capital Accumulation and Factor Prices", NBER WP 6566.
- [4] Blanchard, Olivier (1997), "The Medium Run", *Brookings Papers on Economic Activity*, Fall, 89-141.
- [5] Caballero, Ricardo J. and Mohamad L. Hammour (1998), "Jobless Growth: Appropriability, Factor Substitution, and Unemployment", *Carnegie Rochester Conference on Public Policy*, 48, 51-94.